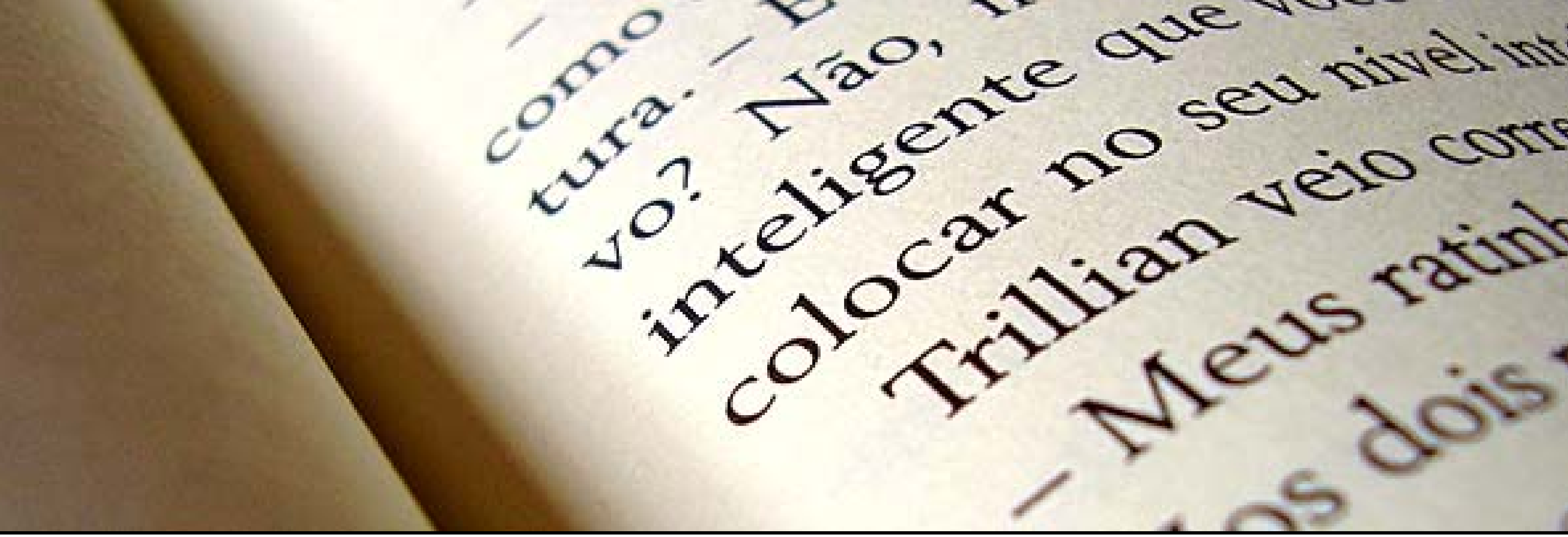

Template-based Paper Reconstruction from a Single Image is Well Posed when the Rulings are Parallel

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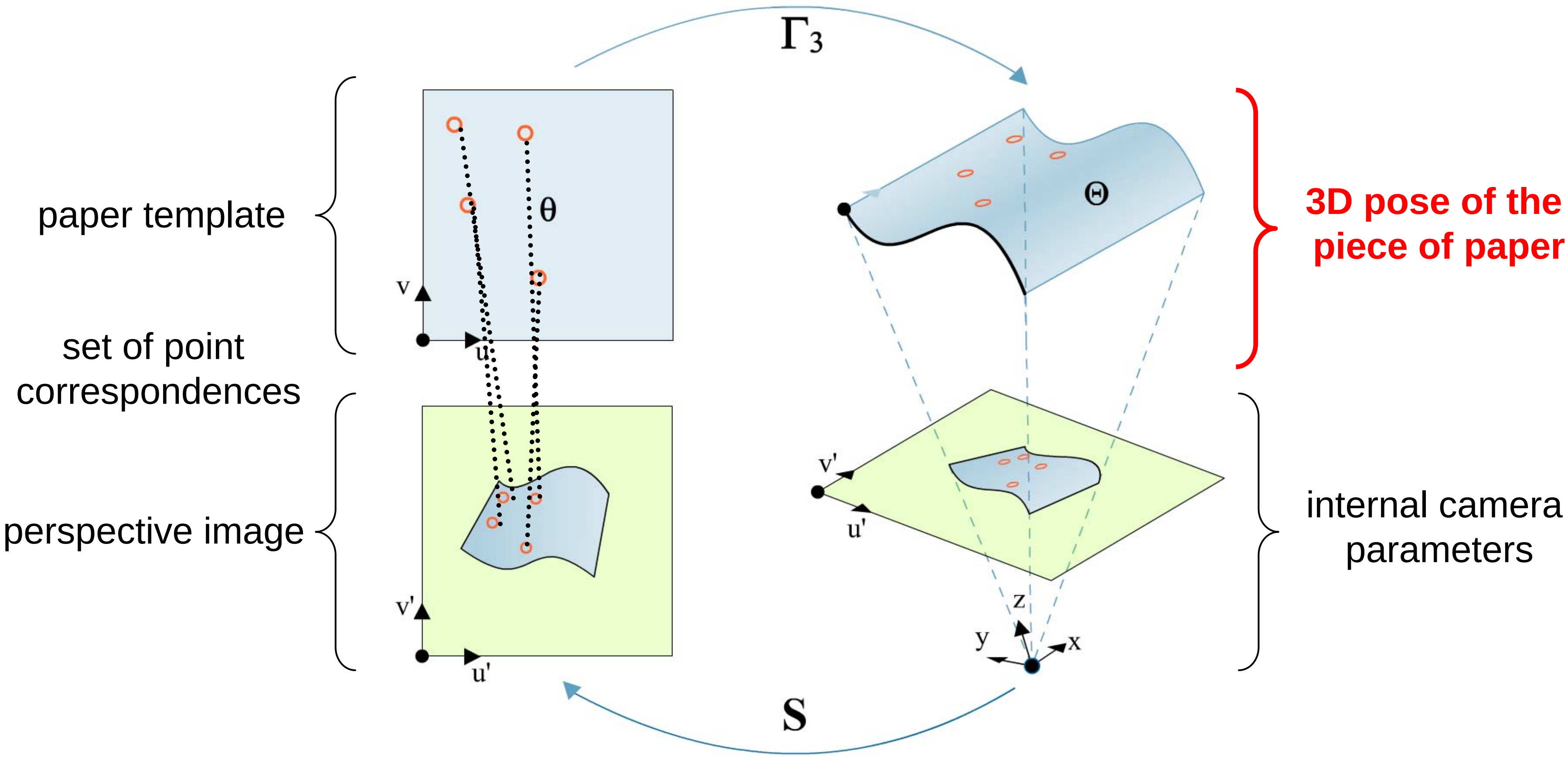
We aim to reconstruct the pose of a piece of paper which is subject to a subset of possible isometries

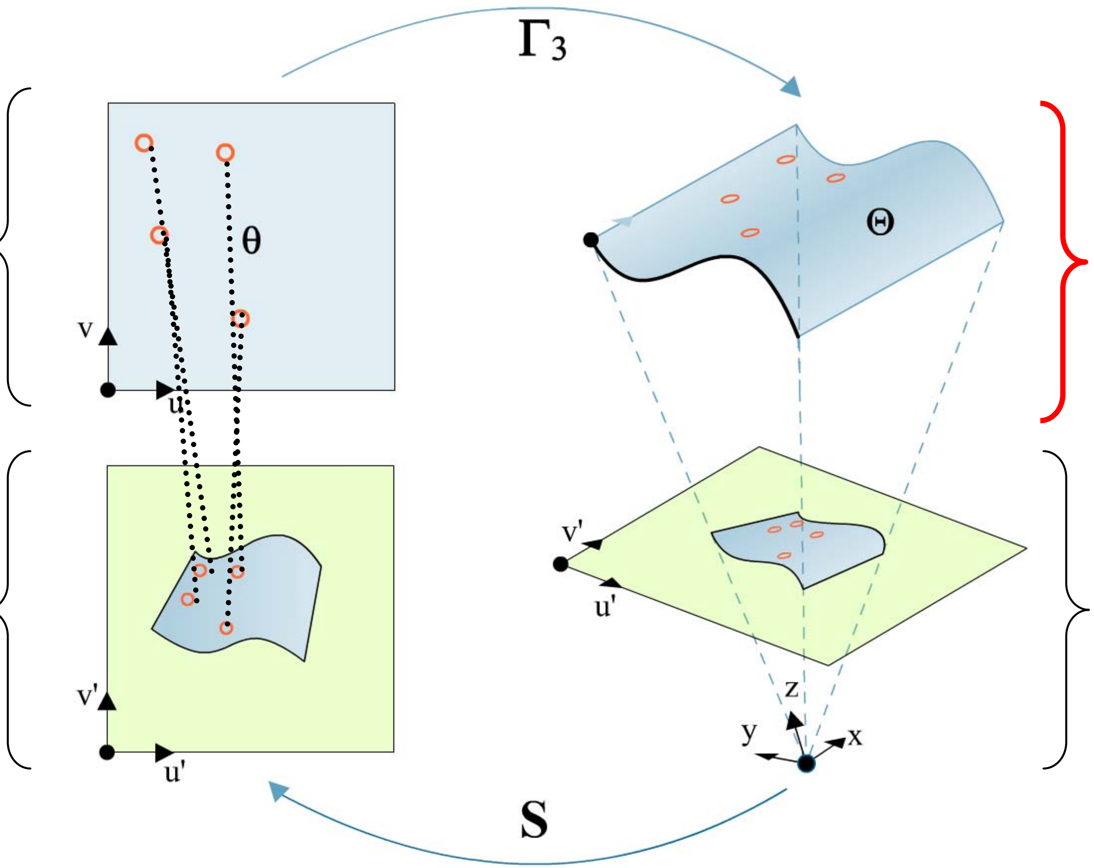


We show that for particular isometries this is a well posed problem

paper template

set of point correspondences

perspective image



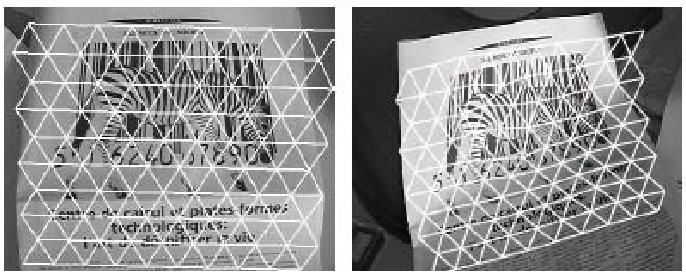
3D pose of the piece of paper

internal camera parameters

Related works

- Template-based monocular deformable surface registration may be performed using general models
  - Generic deformable surfaces using triangular mesh grids

(Julien Pilet, Vincent Lepetit, Pascal Fua)



- Monocular deformable surface reconstruction is possible if some priors are known
  - 3D Morphable Models for face reconstruction

(Volker Blanz and Thomas Vetter)

→ Great works or learnt models to describe real deformations

Related works

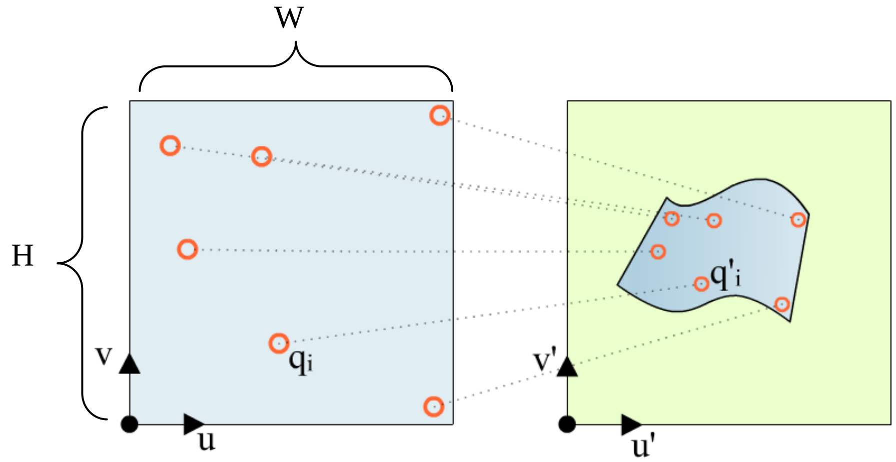
- We address the case of developable surface to model material such as paper
  - Useful for augmentation

- Paper reconstruction may be performed using shape-from-contour
  - mainly for document digitization
    → requires the full knowledge of the contours
  - Not useful in the case of occlusion

a well-posed problem

Assumpions

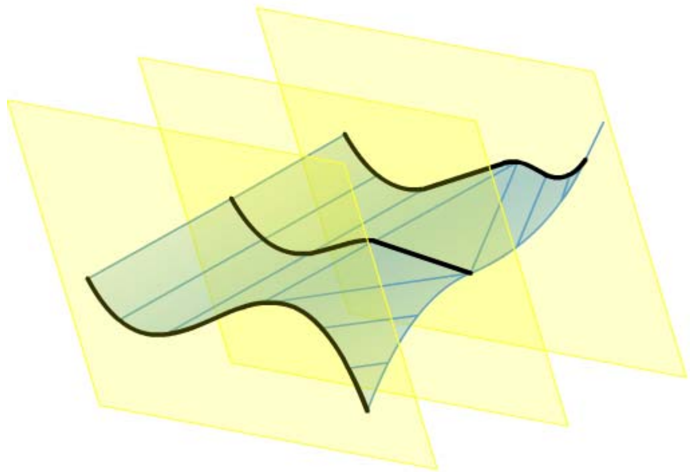
In order to perform a full 3D reconstruction we assume:



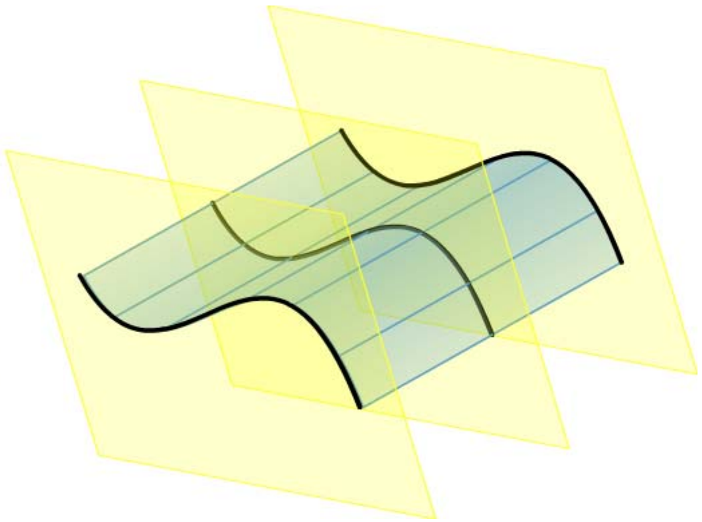
- Set of point correspondences $\{q_i\} \rightarrow \{q'_i\}$
- Internal camera parameters known S
- Metric size of the template (W, H)
- Physical model, developabel surfaces
  - Deformations are **isometries**, thus distances are mantained
  - Vanishing gaussian curvatur

Is the problem well posed?

- The general case of isometric deformations is ill-posed
- We consider a subset of the possible isometries
  - The rulings of the developable surface are constrained to be parallel, i.e. the surface is a generalized cylinder
  - Intuitively this is what happens when book pages are deformed by keeping the binding and the opposite edge parallel.



Generic isometry, ill posed

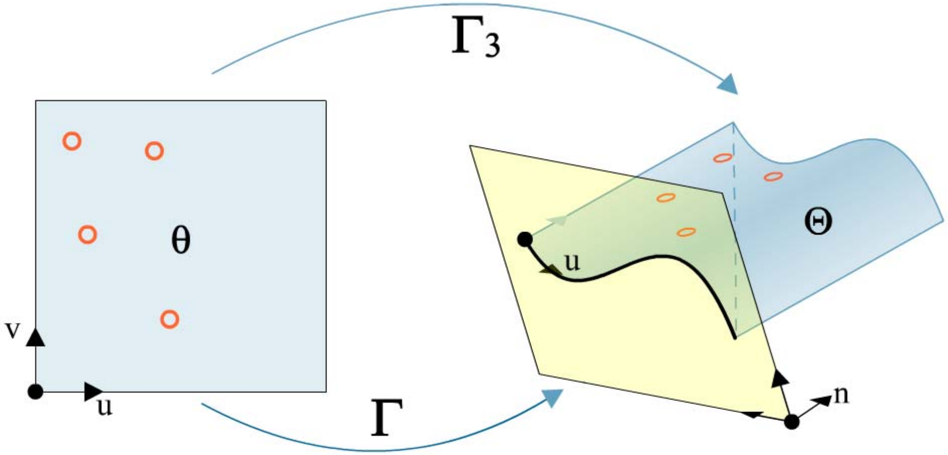


Generalized cylinder, well posed

reduction to a 2D problem

- In the case of a generalized cylinder the surface is parameterized as follows:
  - A generatrix plane π which is perpendicular to all rulings and contains the lower border of the surface
  - A transformation T which maps the XY plane to π, the origin to the bottom left corner, the X axis to the corner segment
  - A $\mathbb{R} \rightarrow \mathbb{R}^2$ mapping Γ which maps u coordinates to a 2D curve on π

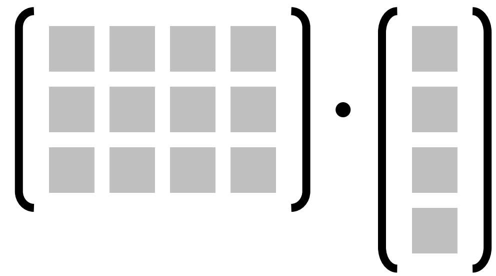
$$\Gamma_3(u, v) = T \begin{pmatrix} \Gamma(u) \\ v \\ 1 \end{pmatrix} \quad \forall u, v \in \theta$$



Reduction to a 2D reconstruction using 1D cameras (1)

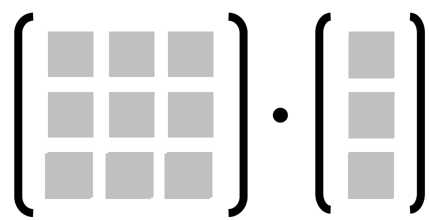
- By considering the projection equation we can derive that:

$$q' \sim S \cdot T \begin{pmatrix} \Gamma(u) \\ v \\ 1 \end{pmatrix}$$



$$q' \sim \begin{pmatrix} s_1 & s_2 & (s_4 + s_3 v) \end{pmatrix} \cdot \begin{pmatrix} \Gamma(u) \\ 1 \end{pmatrix}$$

$$q' \sim \begin{pmatrix} S_{va} \\ S_{vb} \\ S_{vc} \end{pmatrix} \cdot \begin{pmatrix} \Gamma(u) \\ 1 \end{pmatrix}$$



$$q'_u \sim \begin{pmatrix} S_{va} \\ S_{vc} \end{pmatrix} \cdot \begin{pmatrix} \Gamma(u) \\ 1 \end{pmatrix}$$

$$q'_v \sim \begin{pmatrix} S_{vb} \\ S_{vc} \end{pmatrix} \cdot \begin{pmatrix} \Gamma(u) \\ 1 \end{pmatrix}$$

Reduction to a 2D reconstruction using 1D cameras (2)

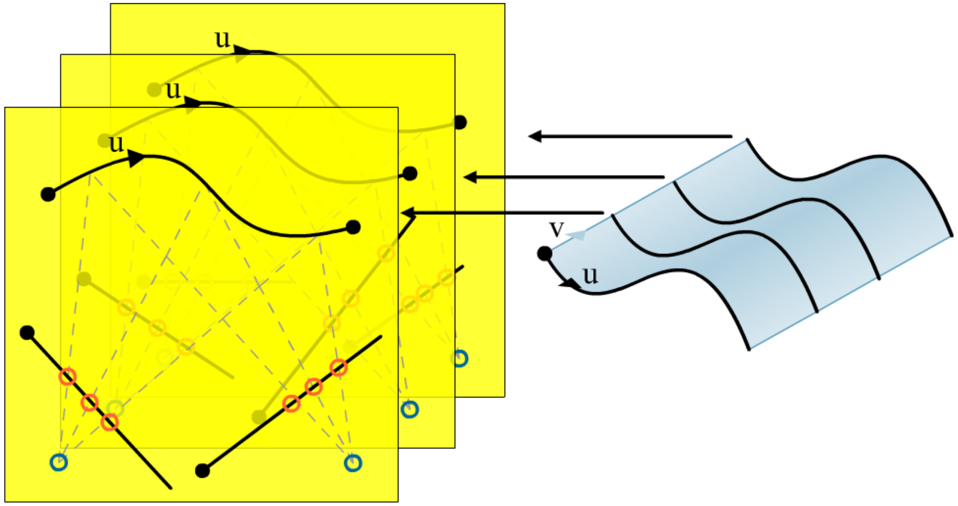
$$q' \sim S \cdot T \begin{pmatrix} \Gamma(u) \\ v \\ 1 \end{pmatrix}$$

$$q'_u \sim \begin{pmatrix} S_{va} \\ S_{vc} \end{pmatrix} \cdot \begin{pmatrix} \Gamma(u) \\ 1 \end{pmatrix}$$

$$q'_v \sim \begin{pmatrix} S_{vb} \\ S_{vc} \end{pmatrix} \cdot \begin{pmatrix} \Gamma(u) \\ 1 \end{pmatrix}$$

- The problem is equivalent to the **reconstruction of 2D points given a pair of 1D cameras** for each surface slice

- u varies the point position over the 2D curve
- v varies the two cameras internal parameters



solving the problem

Formulating the problem (1)

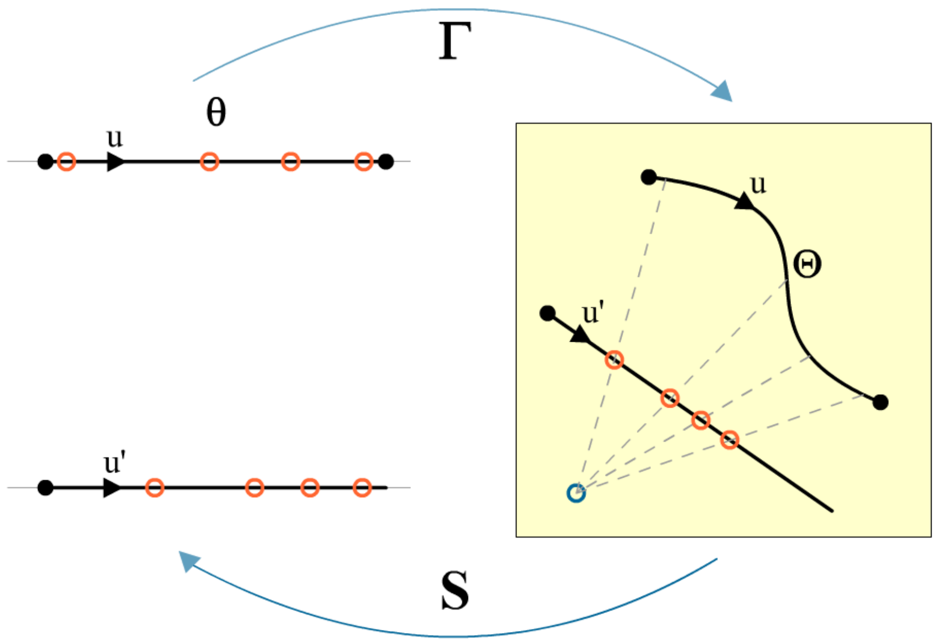
- Isometries preserves gaussian curvature:
  the gaussian curvature is, thus, vanishing everywhere
  - since the parameterization is given by a developable surface this constraint is enforced by construction

- Isometries preserves the metric:
  - By construction distances are preserved along the rulings
  - Since we are assuming a generalized cylinder, if the metric is preserved on π section then it is preserved everywhere

→ Γ must be a 1D isometry

Formulating the problem (2)

- Metric constraints: $\|\Gamma_u\|^2 = 1 \quad \forall u \in \Theta$



- Moreover, we aim to minimize:
  - the reprojection error of the point correspondences
  - a smoothing term

Formulating the problem (3)

- The problem is expressed as a functional optimization:

$$\arg\min_{\Gamma} \left(E_d[\Gamma] + E_m[\Gamma_u] + E_s[\Gamma_{uu}] \right) = \arg\min_{\Gamma} \left(\int e(u, \Gamma, \Gamma_u, \Gamma_{uu}) \right)$$

- This problem depends on the free variable u, function Γ and its **first** and **second derivatives**
- The problem possess natural boundary condition (i.e. the boundary are not fixed)

- The functional E is given by the weighted sum of:
  - $E_d[\Gamma]$: data term, which describe the reprojection error
  - $E_s[\Gamma_{uu}] = \int_\theta \|\Gamma_{uu}(u)\|^2$: smoothing term
  - $E_m[\Gamma_u] = \int_\theta \left(\|\Gamma_u(u)\|^2 - 1 \right)^2$: metric term

Solving the problem

$$\Gamma = \arg\min_{\Gamma} \left(\int e(u, \Gamma, \Gamma_u, \Gamma_{uu}) \right)$$

- This functional optimization is solved by applying the Euler-Lagrange equations
  - this gives a system of PDEs depending up to the fourth derivatives of Γ and a set of PDEs related to the natural boundary condition

- The PDE system is solved using numerical methods:
  - The domain is sampled at N nodes
  - Derivatives are replaced by finite differences approximation

recover the generatrix plane

Generatrix plane recovery (1)

- To exploit the exposed parametrization the generatrix plane transormation is needed
- This can be done by exploiting
  - the template dimensions
  - at least two pair of points on the same ruling, for instance the corners of the largest visible rectangle

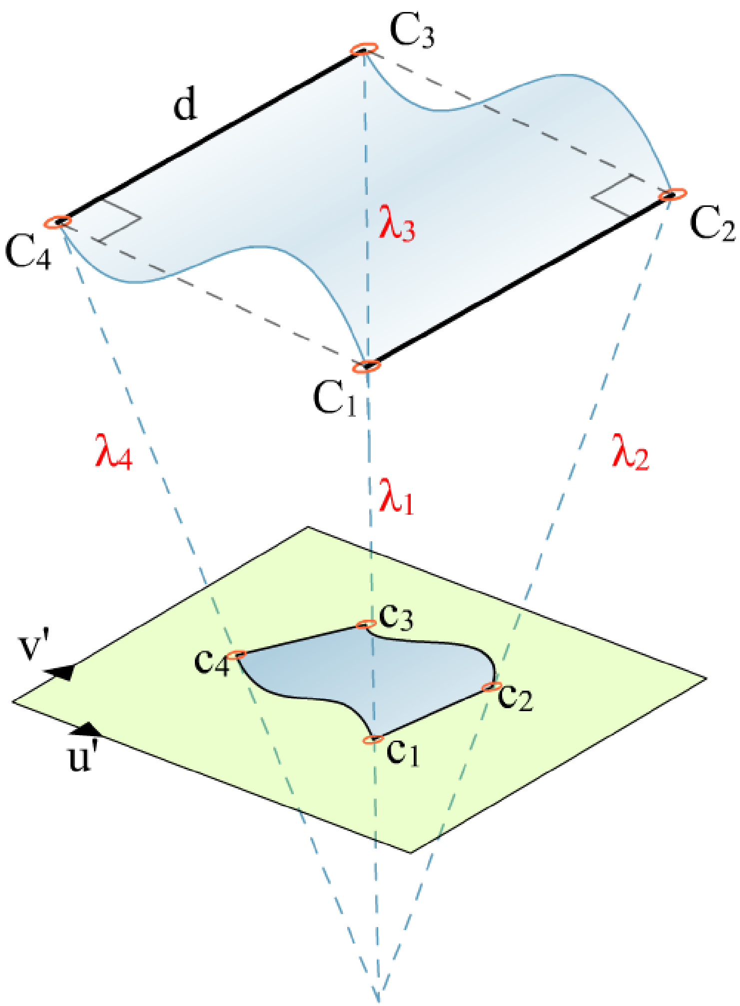
- Using the template dimension the points distances are easily calculated
- We know the camera internal parameter
- The problem can be solved using an optimization procedure

Generatrix plane recovery (1)

- Done exploiting the template dimensions and at least two pairs of points on the same ruling

- Using the template dimension the inter point distances are easily calculated

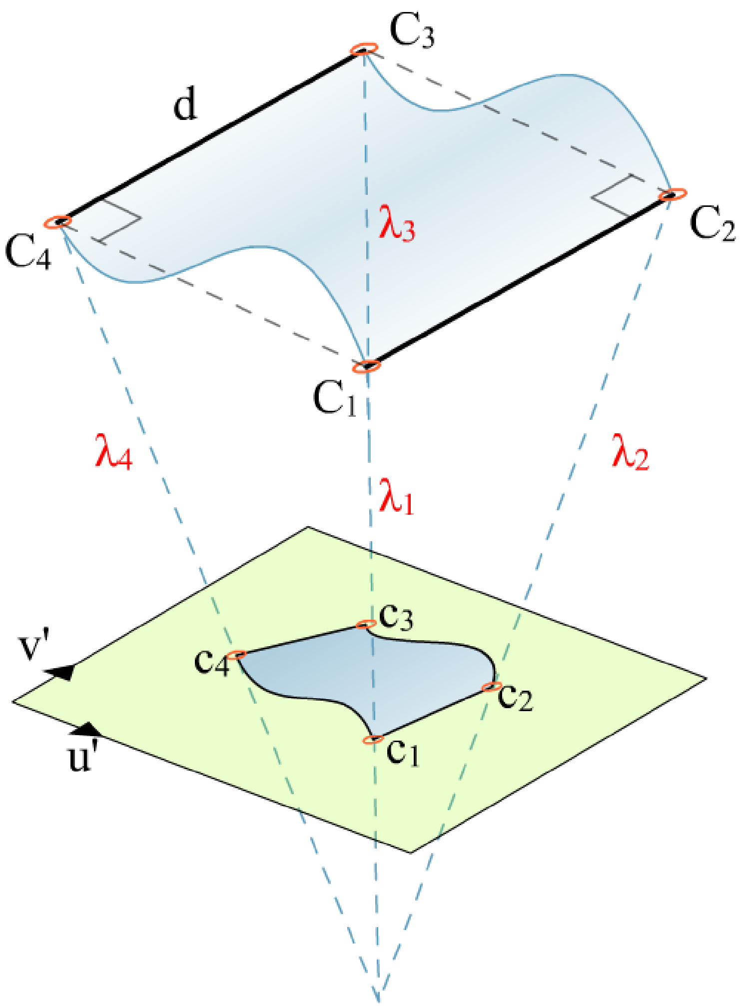
- The problem can be solved using an optimization procedure



- Given the four points $c_{1,}$ c_2 and $c_{3,}$ c_4, the two segments length d and the camera internal parameters

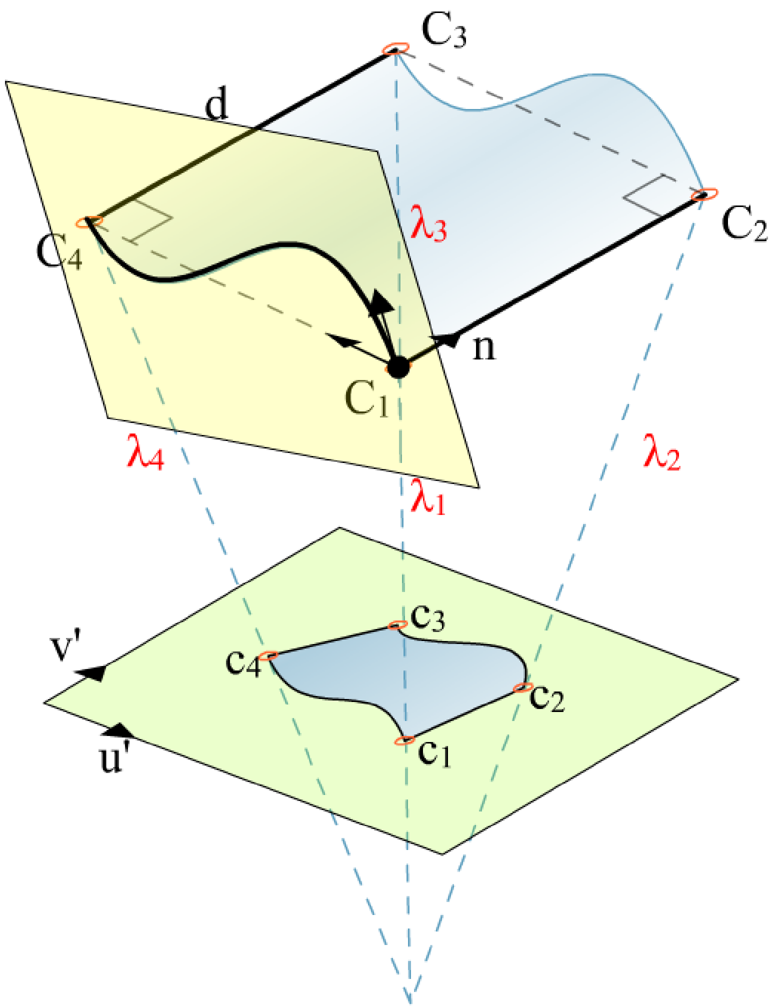
- The unknowns are the four perspective depths λ_1 λ_2 λ_3 and $\lambda 4$

- These may be recovered by enfocing the constriants:
  - C_1C_2 is parallel to $C_3C_{4,}$
  - C_1C_2 is orthogonal to $C_1C_{4,}$
  - C_1C_2 has length d,
  - C_3C_4 has length d



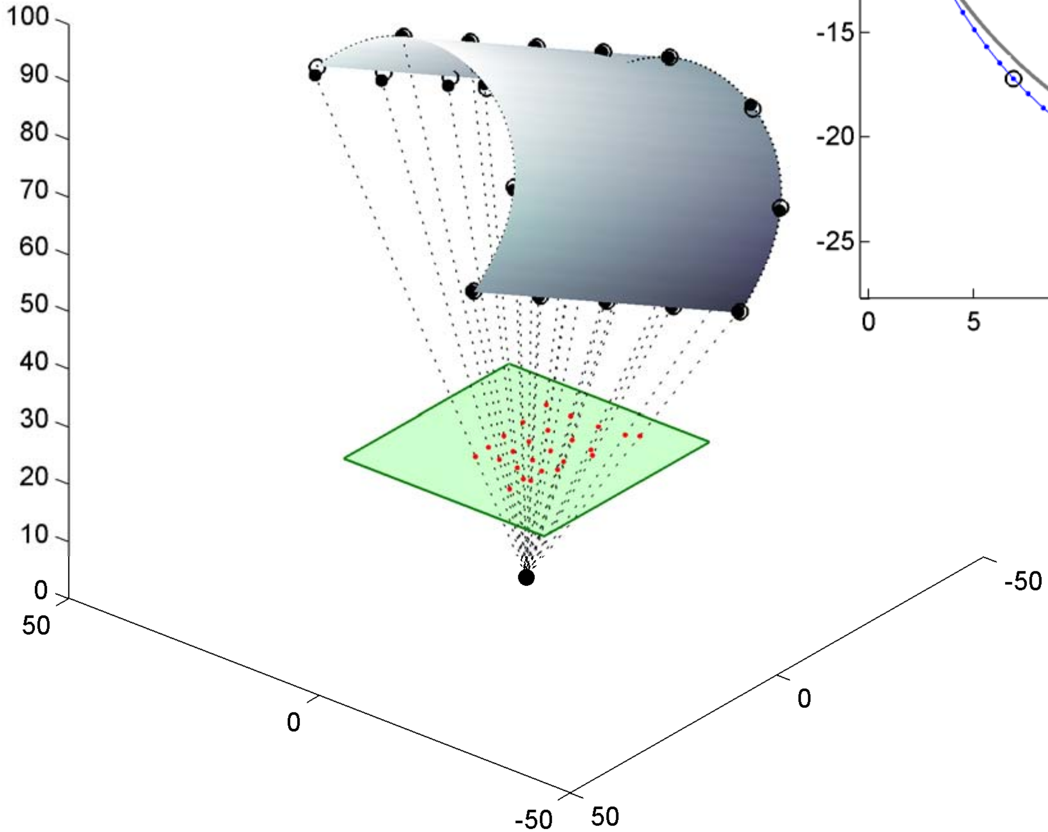
Generatrix plane recovery (3)

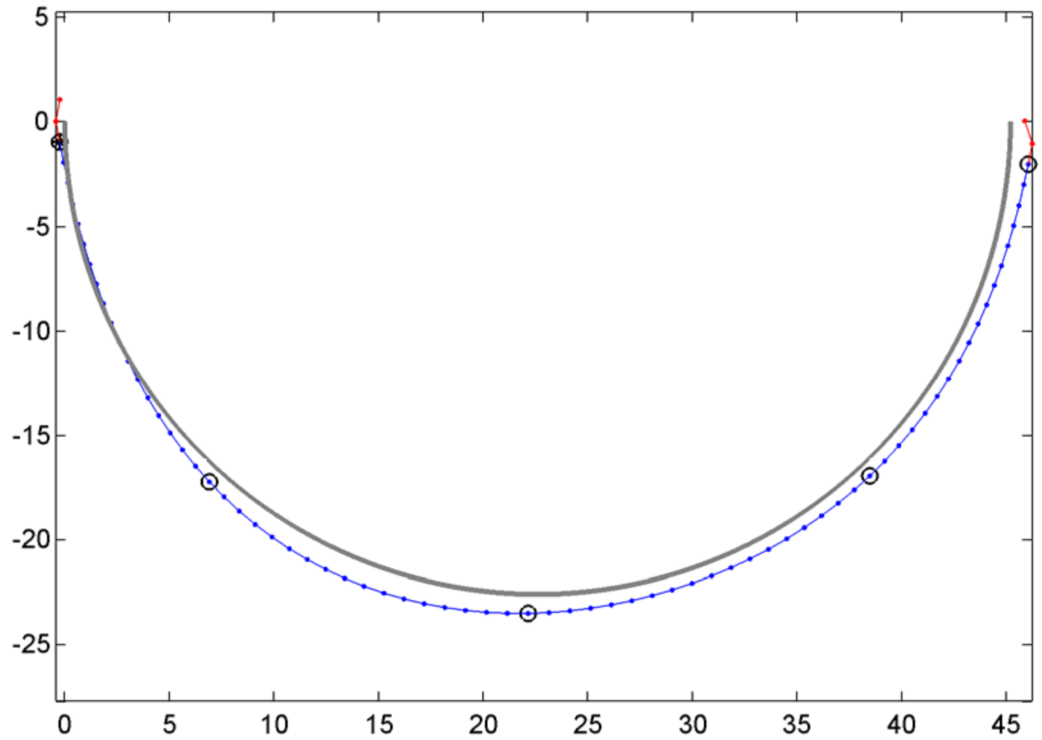
- The generatrix plane π is orthogonal to the plane containing the detected rectangle
- In particular we consider a transformation T which brings
  - The plane XY to π
  - The axis X parallel to C_1C_4,
  - The axis Z parallel to C_1C_2



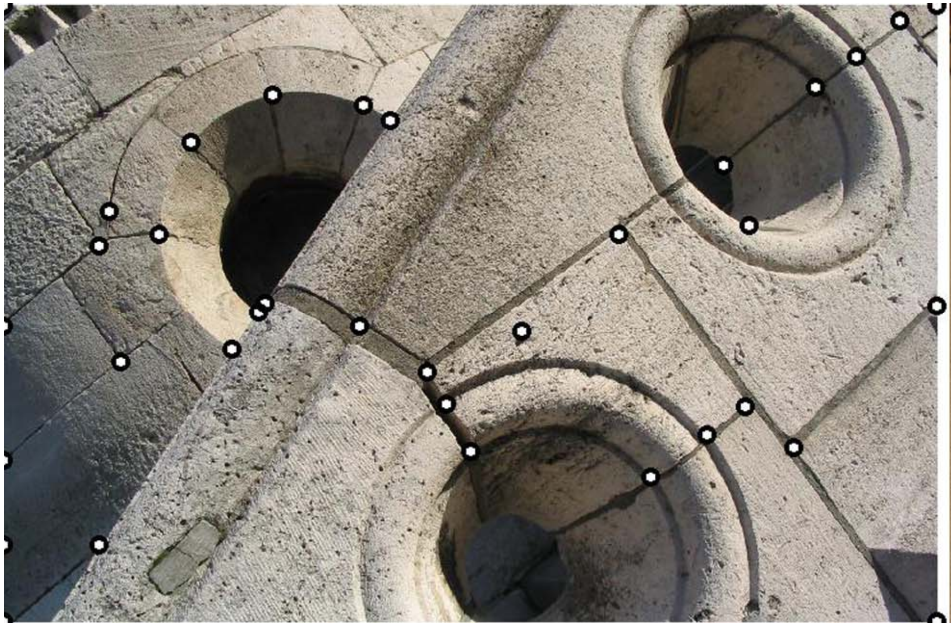
experimental results

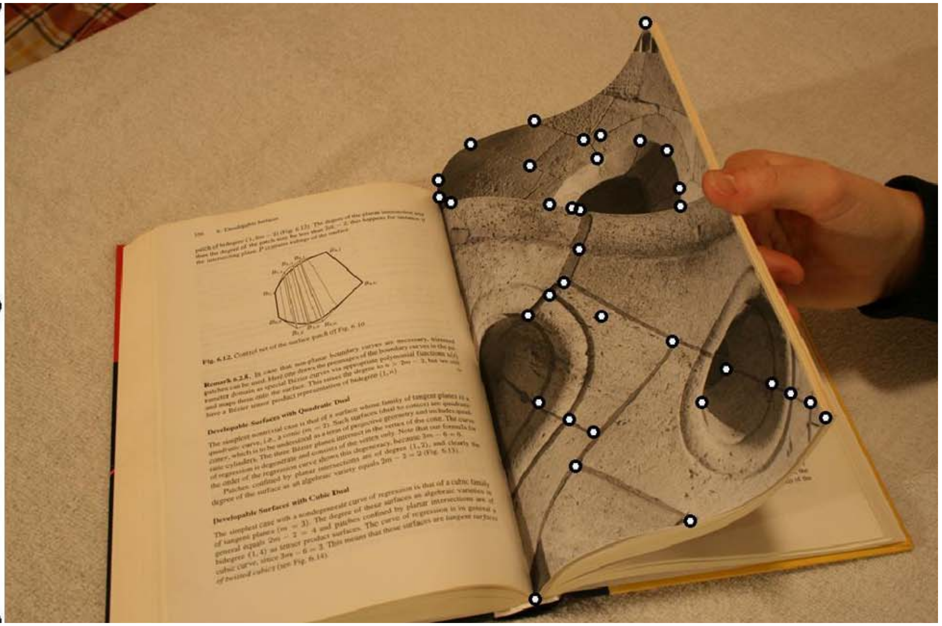
Experimental results



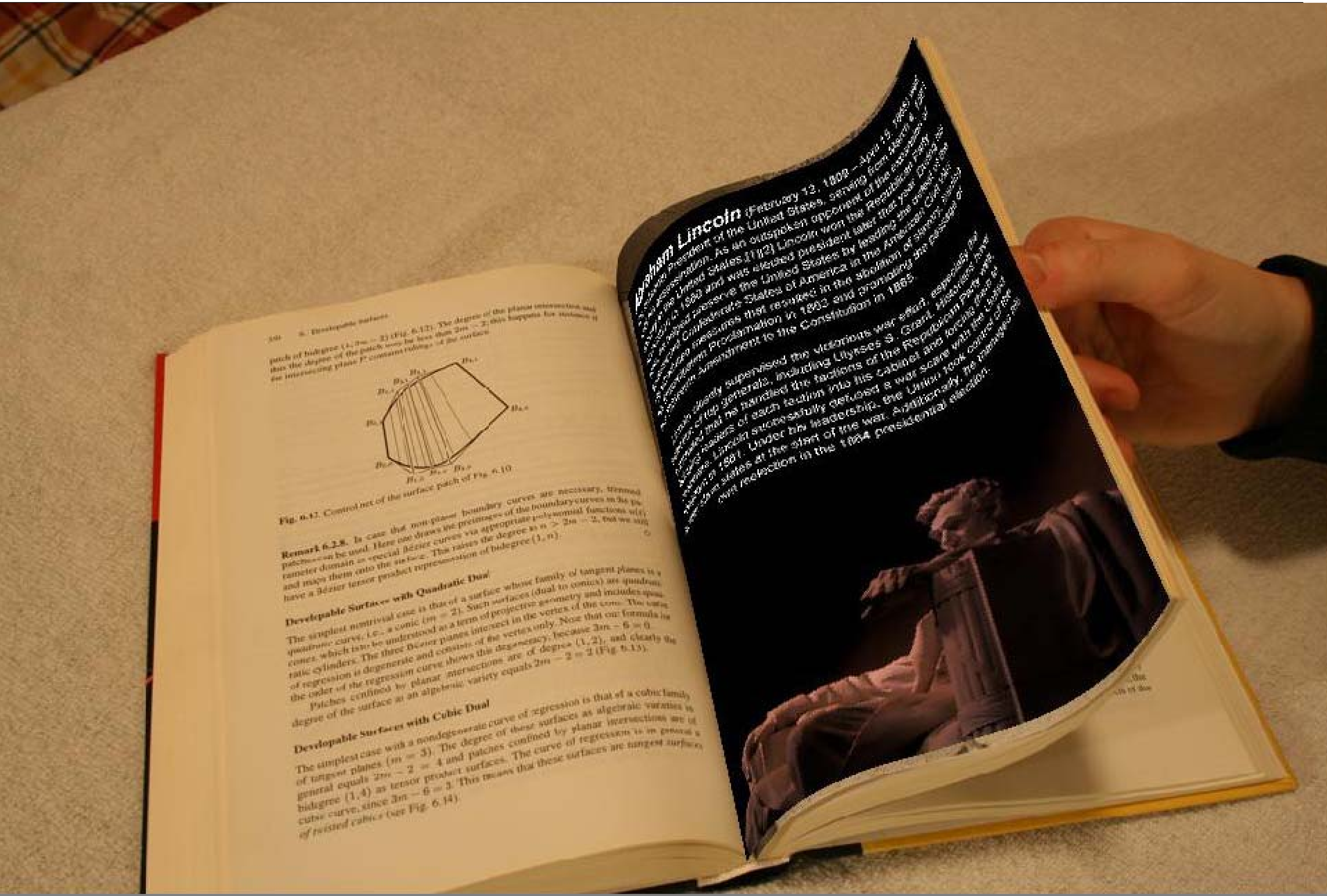


Experimental results

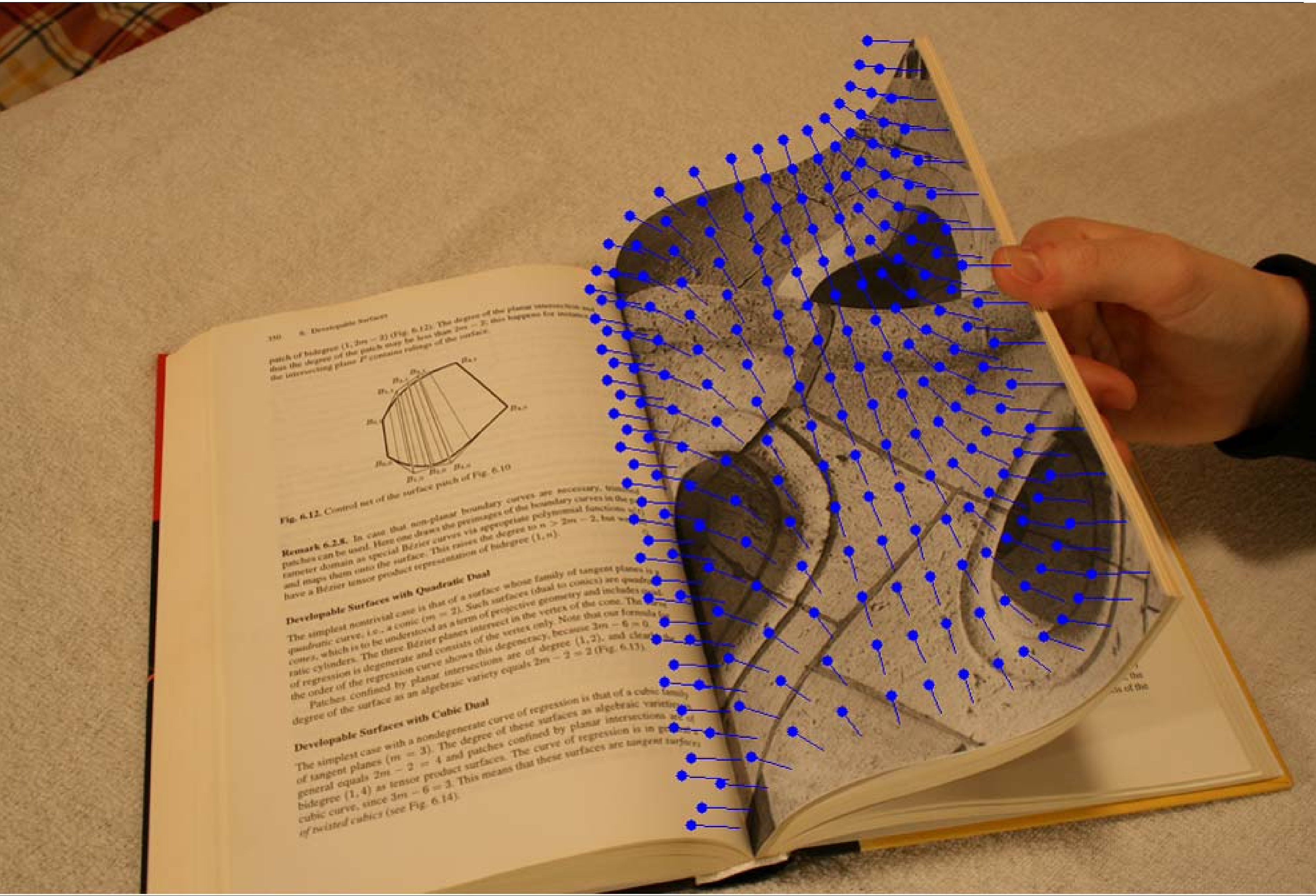




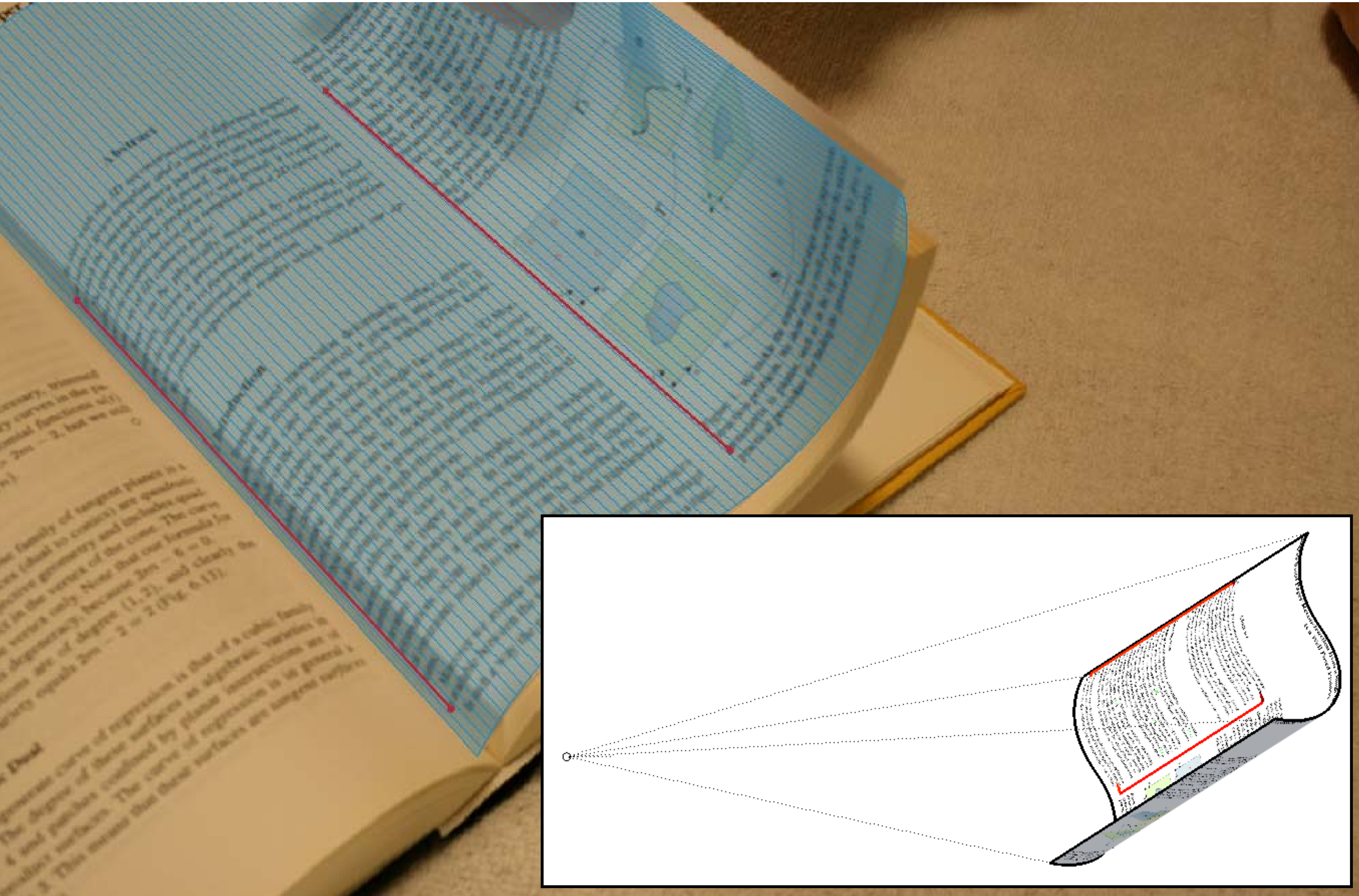
Experimental results (texture replacement)



Experimental results (augmentation)



Experimental results (handling occlusion)



Conclusion and future works

- Template based reconstruction of a generalized cylinder is well posed
- The reconstruction is probably well posed also in the generalized cone case
  - Even if more general, this case is more difficult to reproduce, and the generalized cone parameters are more difficult to recover